Differentially Finite Functions: Exercices

To prepare for Mon 2024-10-21

Exercise 1. Let $f(x) = (1 + x + x^2)^n \in \mathbb{Z}[x]$. Give an algorithm that computes the parity of all coefficients of f in $\mathrm{O}(\mathrm{M}(n))$ **bit** operations.

Exercise 2. Prove the following identity of formal power series:

$$\arcsin(x)^2 = \sum_{k=0}^{\infty} \frac{k!}{\frac{1}{2}\frac{3}{2}\cdots\left(k+\frac{1}{2}\right)} \frac{x^{2k+2}}{2\,k+2}.$$

For this:

1. Check that $y(x) = \arcsin(x)$ is solution to $(1 - x^2)\, y''(x) = x\, y'(x)$.
2. Deduce a linear differential equation satified by $z(x) = y(x)^2$.
3. Deduce a linear recurrence relation satisfied by the coefficients of the series.
4. Conclude.